

UNIVERSITY OF BRISTOL

School of Mathematics

**Ordinary Differential Equations 2**

Test

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20 November 2019

40 minutes

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This paper contains **three** questions. All answers will be used for assessment.

Calculators are not permitted in this examination.

On this examination, the marking scheme is indicative and is intended only as a guide to the relative weighting of the questions.

## 1. (2 marks)

Can an autonomous (one-dimensional) ODE on  $\mathbb{R}$  have a homoclinic orbit? If yes, give an example, if no, explain why not.

## 2. (2 marks)

Give an example of an autonomous (one-dimensional) ODE on  $\mathbb{R}$  which has non-equilibrium orbits, all of which are heteroclinics.

3. (6 marks) Consider the Linear ODE on  $\mathbb{R}^2$ ,

$$\dot{x} = Ax.$$

Here,  $A \in \mathbb{R}^{2 \times 2}$  has  $(1, 1)$  as an eigenvector with eigenvalue 1, and also  $(1, -1)$  as an eigenvector with eigenvalue  $-1$ .

- (a) (2 mark) Characterise the stability of  $(0, 0)$ .
- (b) (2 marks) Draw a phase diagram, clearly indicating the invariant subspaces and their stability.
- (c) (2 marks) Write down the flow. You may leave your answer as a product of matrices (i.e., there is no need to simplify your answer).