UNIVERSITY OF BRISTOL

School of Mathematics

Ordinary Differential Equations 2
Sample Test

November 2019
40 minutes, but designed to be completed in 20 minutes

This paper contains **two** questions. All answers will be used for assessment.

Calculators are not permitted in this examination.

On this examination, the marking scheme is indicative and is intended only as a guide to the relative weighting of the questions.

1. (**5 marks**)

   Consider the following ODE for $(x, y) \in \mathbb{R}^2$:

$$\begin{aligned} \dot{x} &= x\ f(x, y), \\ \dot{y} &= y\ g(x, y), \end{aligned}$$

   where $f, g \colon \mathbb{R}^2 \to \mathbb{R}$ are smooth functions.

   Show that each of the four quadrants, i.e. each of the sets

$$\begin{aligned} &\left\{(x, y) \in \mathbb{R}^2 \mid x > 0, y > 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x > 0, y < 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x < 0, y > 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x < 0, y < 0\right\}, \end{aligned}$$

   is an invariant set.

2. (**10 marks**)

Which of the following trajectories on $\mathbb{R}^2$ can be solutions of a first-order autonomous linear ODE?

(a) $(x(t), y(t)) = (3e^t + e^{-t}, e^{2t})$.

(b) $(x(t), y(t)) = (3e^t + e^{-t}, e^t)$.

(c) $(x(t), y(t)) = (3e^t + e^{-t}, te^t)$.

(d) $(x(t), y(t)) = (3e^t, t^2e^t)$.

(e) $(x(t), y(t)) = (e^t + 2e^{-t}, e^t + 2e^{-t})$.

You must justify your answer.

End of examination.