UNIVERSITY OF BRISTOL

School of Mathematics

Ordinary Differential Equations 2

Mock Exam 2

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17 December 2018 45 minutes

This paper contains **four** questions. All answers will be used for assessment.

Calculators are not permitted in this examination.

On this examination, the marking scheme is indicative and is intended only as a guide to the relative weighting of the questions.

1. (**8 marks**)

   Consider the following ODE for $(x, y) \in \mathbb{R}^2$:

   $$\dot{x} = y - x\ f(x, y),$$
   $$\dot{y} = -x - y\ f(x, y),$$

   where $f \colon \mathbb{R}^2 \to \mathbb{R}$ has a Taylor expansion around $(0, 0)$ and satisfies $f(0, 0) = 0$.

   (a) (**4 marks**) Show that $(0, 0)$ is stable if f is non-negative in a neighbourhood around the origin.

   (b) (**4 marks**) Under what conditions on f is $(0, 0)$ asymptotically stable?

2. (**10 marks**)

   Consider the following ODE for $(x, y) \in \mathbb{R}^2$:

   $$\dot{x} = -y - x^3,$$
   $$\dot{y} = x^5.$$

   Use the function $V(x, y) = x^6 + 3y^2$ to show that all solutions of the ODE approach $(0, 0)$ as $t \to \infty$.

3. (**4 marks**)

   How many periodic orbits exist for the ODE

   $$\dot{x} = xe^{-x},$$
   $$\dot{y} = 1 + x + y^2,$$

   where $(x, y) \in \mathbb{R}^2$? You must justify your answer.

4. (**8 marks**)

   Consider the following ODE for $x \in \mathbb{R}$:

   $$\dot{x} = \mu x - \sin x,$$

   where $\mu \in \mathbb{R}$ is a parameter.

   (a) (**3 marks**)
   Classify the bifurcation at $\mu = 1$.

   (b) (**5 marks**)
   Identify the largest $\mu < 1$ for which the system has a bifurcation. What kind of a bifurcation is it?

End of examination.