UNIVERSITY OF BRISTOL

School of Mathematics

Ordinary Differential Equations 2
Mock Exam

9 November 2018 45 minutes

This paper contains **three** questions. All answers will be used for assessment.

Calculators are not permitted in this examination.

On this examination, the marking scheme is indicative and is intended only as a guide to the relative weighting of the questions.

1. (**15 marks**)

   Consider the following ODE for $(x, y) \in \mathbb{R}^2$:

   $$\begin{aligned} \dot{x} &= \lambda x, \\ \dot{y} &= x + \lambda y, \end{aligned}$$

   where $\lambda > 0$.

   (a) (**4 marks**)
   Determine the equilibrium point(s) and its/their stability.

   (b) (**6 marks**)
   Compute the flow generated by the ODE.

   (c) (**5 marks**)
   Draw the phase portrait.

2. (**5 marks**)

   Consider the following ODE for $(x, y) \in \mathbb{R}^2$:

   $$\begin{aligned} \dot{x} &= x\ f(x, y), \\ \dot{y} &= y\ g(x, y), \end{aligned}$$

   where $f, g \colon \mathbb{R}^2 \to \mathbb{R}$ are smooth functions.

   Show that each of the four quadrants, i.e. each of the sets

   $$\begin{aligned} &\left\{(x, y) \in \mathbb{R}^2 \mid x > 0, y > 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x > 0, y < 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x < 0, y > 0\right\}, \\ &\left\{(x, y) \in \mathbb{R}^2 \mid x < 0, y < 0\right\}, \end{aligned}$$

   is an invariant set.

3. (**10 marks**)

   Which of the following trajectories on $\mathbb{R}^2$ can be solutions of a first-order autonomous linear ODE?

   (a) $(x(t), y(t)) = (3e^t + e^{-t}, e^{2t})$.
   (b) $(x(t), y(t)) = (3e^t + e^{-t}, e^t)$.
   (c) $(x(t), y(t)) = (3e^t + e^{-t}, te^t)$.
   (d) $(x(t), y(t)) = (3e^t, t^2e^t)$.
   (e) $(x(t), y(t)) = (e^t + 2e^{-t}, e^t + 2e^{-t})$.

   You must justify your answer.

End of examination.