Mathematics Examination Feedback Form

This form is intended to provide generic feedback to students on examination performance in individual units, in line with university code of practice for the assessment of taught programmes. Its purpose is to help students to develop their skills, knowledge and understanding and help them evaluate their current level of performance.

Unit Title:	Ordinary Differential Equations 2
Unit Code:	MATH20101
Examination Markers:	Isaac Chenchiah (Q1 & Q4) & Charles Harris (Q2 & Q3)

Common areas that were well done:
Q1a. The majority of students were able to verify that the given flow satisfied the ODE and to compute the stable and unstable manifolds. Q1b. Fewer, but still many, students were able to compute the global stable and centre manifolds. Q2. The majority of students were able to compute the stability of the equilibrium at (0,0) [part (a)], to show that V(x,y) is constant along the trajectories [part (b)] and were able to identify heteroclinic orbits [part (f)]. Q3. The majority of students were able to identify the equilibria [part (a)], compute their stability [part (b)], show that there are non-equilibrium periodic orbits using Bendixon's criterion [part (c)] and determine correctly that \alpha > 0 is the solution of part (e). Q4.Almost all students were able to describe the bifurcation though many failed to realised that the radial coordinate must be non-negative.

Common errors, misunderstandings or other areas requiring improvement:
Q1a. Few students noticed that the given flow could be differentiated and thus verified as a solution to the ODE. Q1b. Some students were unable to distinguish between the stable/unstable/centre subspaces of the linearised system and the corresponding manifolds for the nonlinear system.

Q2.

Stability of the (0,0) [part (a)] Although almost all students established the stability and type of equilibrium that (0,0) is, their reasons related to the linearised system. So only very few noted that, as the equilibrium is hyperbolic the same can be said of the nonlinearised system. (Same comment for Q3(b).)

The phase portrait [part (c)] Not much success on this. Note that here you can approach this in two ways.

- The trick way. You know that V(x,y) is constant along trajectories. So the trajectories trace out the level curves of V(x,y). Identify these. Identify the vectors (\dot{x},\dot{y}) in appropriate parts of the plane - for example along the x-axis will suffice - to put the directional arrows on the phase diagram.
- The standard way. Work out what type each of the three equilibria is. Then work out the what the vector field looks like on the axes as suggested above (or along other vertical or horizontal lines).

Homoclinic orbits [part (d)] Some confusion here.

- Homoclinic orbits start and end at the same equilibrium point. A periodic orbit around an equilibrium point is NOT a homoclinic orbit.
- Some students pointed out the TRIVIAL homoclinic orbits, i.e. the equilibrium points themselves. Since this is allowed for in the definition in the printed notes this solution is acceptable (but clearly not the one the exam was looking for). However it is false to state that only the stable equilibria are this type of orbit – there was a clear misunderstanding here on a number of scripts – in that stability describes what happens close to the equilibrium point but NOT what happens at the point itself (which, being an equilibrium stays put forever…).

Periodic orbits [part (e)] Some confusion here also.

- Firstly a homoclinic orbit is NOT periodic (make sure that you understand this).
- Secondly a number of students drew nice circular orbits around single equilibria, and other closed curves around all three equilibria on their phase portrait, but did not recognise that, for a two dimensional phase portrait these are precisely the periodic orbits that you are looking for.

Q3.

Using LaSalles Invariance Principle [part (d)] Varied results but clearly not fully understood by a significant number of students. The following relate to some misunderstandings.

- You cannot take \mathcal{M} to be the whole of R^2 as this is NOT compact.
- Although a majority of students used as their compact set \mathcal{M} of the form {(x,y) : V(x,y) <= c} – i.e. the set of elements enclosed by the level curve {(x,y) : V(x,y) = c} in R^2 - there was often not a clear indication of the following:

1. The graph of V(x,y) looks like a paraboloid in R^3 except for the bottom part which has a couple of humps and (circular) bumps. So only if you take c large enough does the level curve corresponding to c define some sort of (a bit irregular as in the outer curves of the phase portrait of Q) ellipse so that \mathcal{M} := {(x,y) : V(x,y) <= c} is compact (and contains all the points drawn out by the level curve {(x,y) : V(x,y) = c} in the plane R^2). If c is too small then this set is not necessarily compact (and also does not necessarily contain all three equilibria).
2. You should remember that c needs to be large enough so that the tangent vector u to V(x,y) along the whole of the level curve defined by c points up and outwards. In this case, as \dot{V}(x,y) is the dot product of u with (\dot{x},\dot{y}), if the latter also pointed outwards \dot{V}(x,y) would be positive. Since \dot{V}(x,y) <= 0 (as you will have shown) you must have that either (\dot{x},\dot{y}) points inwards or along the curve itself. Hence \mathcal{M} := {(x,y) : V(x,y) <= c} is positively invariant. A significant number of students omitted to state the requirement that c be

large enough for positive invariance to kick in – and clearly had not fully understood this part of Lasalle's result (which was discussed in detail with those who came to the feedback classes).

3. A number of students defined \mathcal{M} correctly but then defined the set E as being the set of points (x,y) in R^2 for which V(x,y) = 0 and NOT the set of such points contained in \mathcal{M}.

Q4b.

Only about 5 students solved this question correctly in its entirety, though many received partial credit. For many students the primary obstacle was a complete lack of familiarity with polar coordinates even though this was covered in Calculus 1 and in several examples in the lectures and problems classes.