ODE 2, Problem Set 9 Solutions

1. $$\dot{x} = \mu x - 3y - x(x^2+y^2)^3$$
$$\dot{y} = 3x + \mu y - y(x^2+y^2)^3$$

$(x, y) = (0, 0)$ is a fixed point.

The eigenvalues of the matrix associated with the linearization about this fixed point are

$$\lambda_{1,2} = \mu \pm 3i$$

So it is not hyperbolic at $\mu = 0$.

Transform the vector field to polar coordinates:

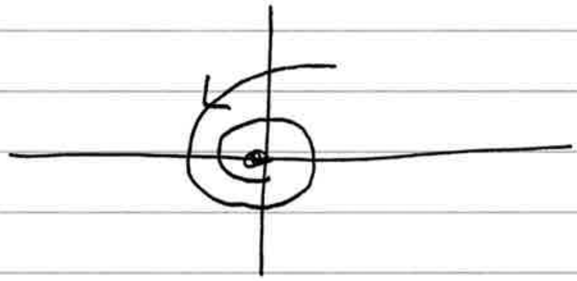
$$x = r\cos\theta, \qquad y = r\sin\theta$$

$$\dot{r} = \mu r - r^7$$
$$\dot{\theta} = 3$$

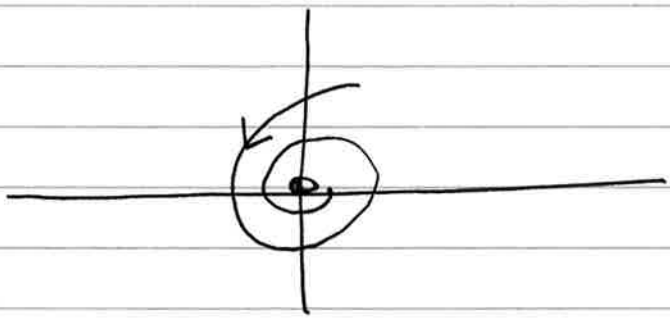
Analyze the bifurcation of the $\dot{r}$ component.

$$\dot{r} = r(\mu - r^6)$$

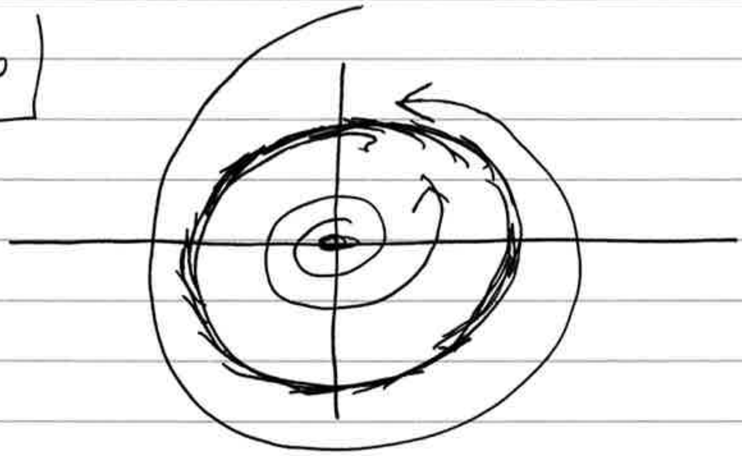
$\mu < 0$



$\mu = 0$

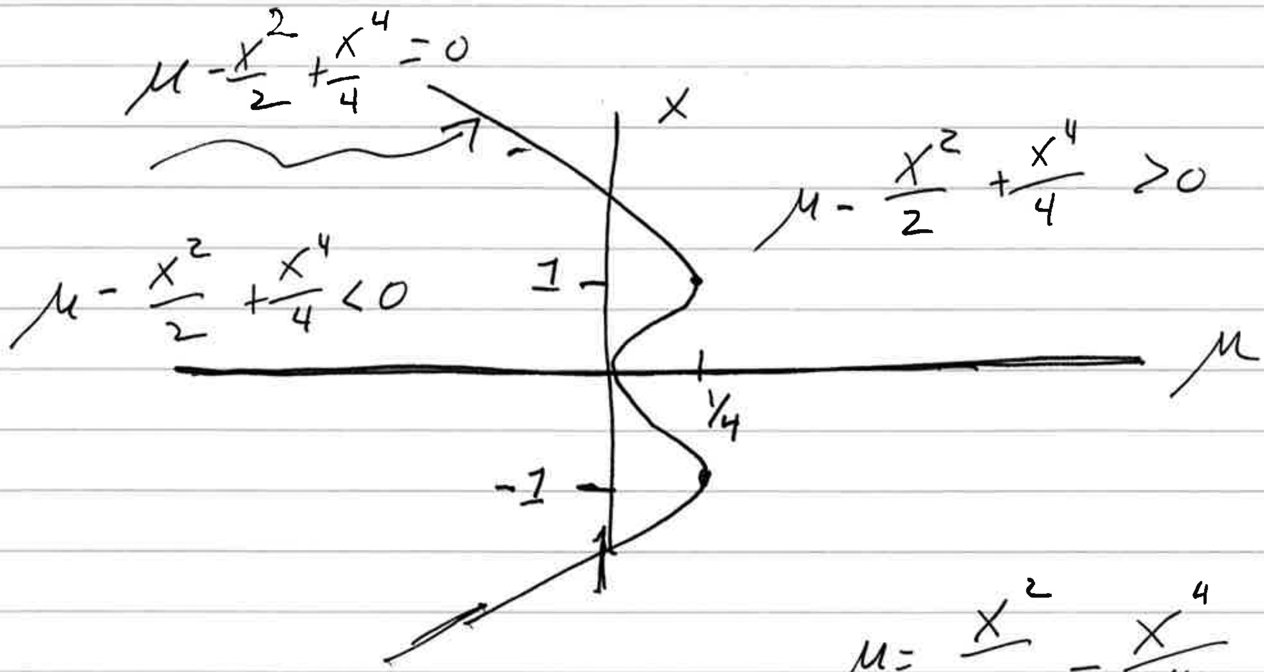


$\mu > 0$



$$\dot{x} = \mu x - \frac{x^3}{2} + \frac{x^5}{4}$$

$$= x\left(\mu - \frac{x^2}{2} + \frac{x^4}{4}\right)$$



$$\mu = \frac{x^2}{2} - \frac{x^4}{4}$$

$$\frac{d\mu}{dx} = x - x^3 = x(1-x^2)$$

μ at $x=1$ $= \frac{1}{4}$

Saddle-node bifurcations occur at $\mu = 1/4$

A pitchfork bifurcation occurs at $\mu = 0$

stability