Chapter 8

1. $$\dot{x} = \mu + 10x^2$$
$$\dot{y} = x - 5y$$

Fixed points.

$$\mu + 10x^2 = 0 \Rightarrow x = \pm\sqrt{\frac{-\mu}{10}}$$

$$y = \frac{x}{5} = \pm\frac{1}{2}\sqrt{\frac{-\mu}{10}}$$

$\Rightarrow$ 2 fixed points for $\mu < 0$

0 fixed points for $\mu > 0$.

Saddle - node bifurcation.

linearized stability of the fixed points.

Matrix associated with the linearization (Jacobian).

$$A = \begin{pmatrix} 20x & 0 \\ 1 & -5 \end{pmatrix}$$

$$\det A = -100x$$

$\det A < 0$ at $x = \sqrt{\frac{-\mu}{10}}$ $\Rightarrow$ saddle

$\det A > 0$ at $x = -\sqrt{\frac{-\mu}{10}}$ $\Rightarrow$ sink

2. $$\dot{x} = \mu x + 10x^2$$
$$\dot{y} = x - 2y$$

Fixed points

$x(\mu + 10x) = 0$, $\quad x = 0$, $\quad x = \frac{-\mu}{10}$

$y = \frac{x}{2} \Rightarrow y = 0$, $\quad y = -\frac{\mu}{20}$

$(x, y) = (0, 0)$, $\left(\frac{-\mu}{10}, \frac{-\mu}{20}\right)$

Linearized Stability

$$\begin{pmatrix} \mu + 20x & 0 \\ 1 & -2 \end{pmatrix}$$

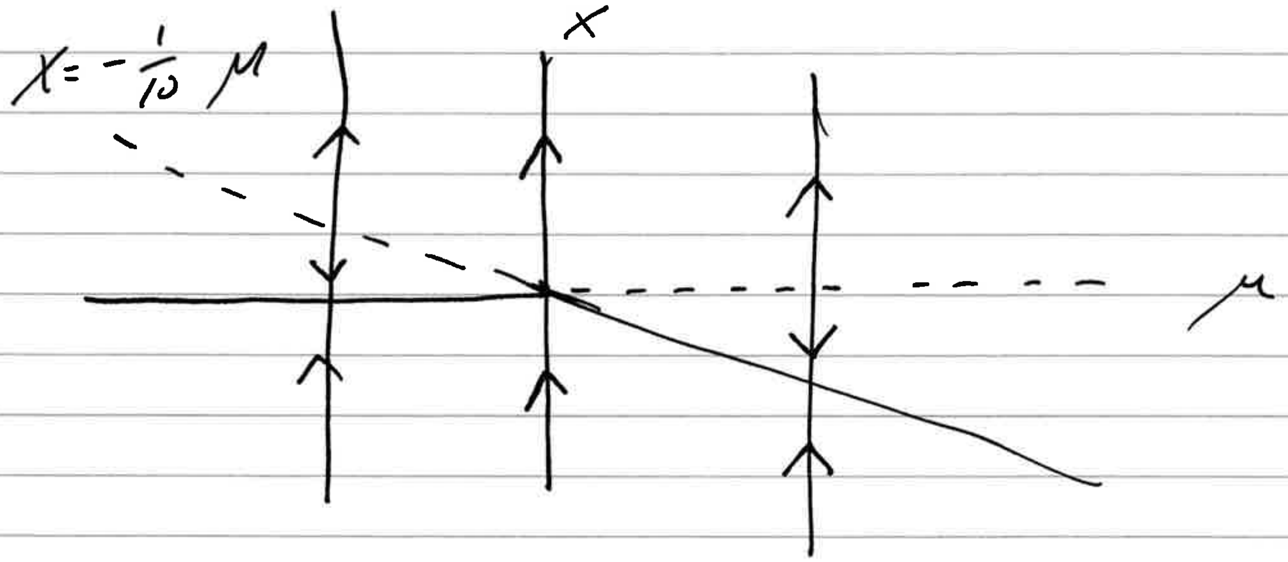
$\boxed{(0,0)}$

$$\lambda_{1,2} = \mu, \quad -2$$

$\boxed{\left(-\frac{\mu}{10}, \frac{-\mu}{20}\right)}$

$$\lambda_{1,2} = -\mu, \quad -2$$

$$\dot{x} = x(\mu + 10x)$$



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3. $\dot{x} = \mu x + x^5$

$\dot{y} = -y$

Fixed points

$x(\mu + x^4) = 0$

$x = 0$, $\mu = -x^4$, or $x = (-\mu)^{1/4}$

$y = 0$

$(x, y) = (0, 0)$, $((-\mu)^{1/4}, 0)$

$\dot{x} = x(\mu + x^4)$