3. $$\dot{x} = y$$
$$\dot{y} = x - x^3 - \alpha x^2 y, \quad \alpha > 0$$

Fixed points

$$(x, y) = (0,0), \ (\pm 1, 0)$$

Let

$$V(x,y) = \frac{y^2}{2} - \frac{x^2}{2} + \frac{x^4}{4}$$

$$\dot{V} = (-x + x^3, \ y) \cdot (y, \ x - x^3 - \alpha x^2 y)$$

$$= -\alpha x^2 y^2$$

The curve bounded by $V = c$, for $c$ larger than $1$, bounds a positive invariant set ($M$)

$$E = \{ (x,y) \in \mathbb{R}^2 \mid \dot{V} = 0 \}$$
$$= \{ (x,y) \in \mathbb{R}^2 \mid x = 0 \text{ or } y = 0 \}$$

Points on the y axis (except $y=0$) leave the y axis since $\dot{x} \neq 0$

Points on the x axis (except $x = \pm 1, 0$) leave the y axis since $\dot{y} \neq 0$ except at these points.

The set of points that are in E and remain in E for all time

$$M = \{ (0,0), (\pm 1, 0) \}$$

Then by the La Salle invariance principle all trajectories approach one of thes fixed points as $t \to \infty$.

4. $\dot{x} = y$

$\dot{y} = x - x^3 + \alpha x y$

$$\left\| \text{Equilibrium points, matrix associated with the equilibrium point, and eigenvalues of the matrix} \right\|$$

$(0,0)$ : $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ; $\lambda_{1,2} = \pm 1$

$(1,0)$ : $\begin{pmatrix} 0 & 1 \\ -2 & \alpha \end{pmatrix}$ ; $\lambda_{1,2} = \frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 8}$

$(-1,0)$ : $\begin{pmatrix} 0 & 1 \\ -2 & -\alpha \end{pmatrix}$ ; $\lambda_{1,2} = -\frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 8}$

$\boxed{(0,0)}$ is a saddle for all values of $\alpha$.

$$\alpha^2 - 8 < \alpha^2$$

So the first term in the expression for the eigenvalue determines stability.

$\boxed{(1,0)}$ For $\alpha = 0$ the eigenvalues are purely imaginary.

As $\alpha$ goes from negative to positive $(1,0)$ goes from a sink to a source.

$\boxed{(-1, 0)}$

As $\alpha$ goes from negative to positive $(-1, 0)$ goes from a source to a sink.